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THEORETICAL STUDIES OF PROTONS IN THE OUTER RADIATION BELT

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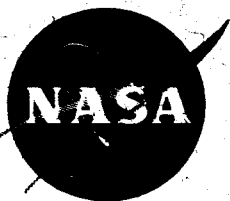
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Theoretical Studies of Protons in the Outer
Radiation Belt

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ABSTRACT

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The variations in the energy spectra with pitch angle and L of the relatively stable 0.1 to 5 Mev protons in the outer radiation belt have been found to be in good agreement with the results of a model that permits rapid motion of the protons in L space. In this model, the protons violate the third adiabatic invariant of trapped particle motion but do not violate the first two adiabatic invariants. Changes in fluxes with L are not consistent with Liouville's Theorem. Both the departure from Liouville's Theorem and variations in energy spectra seem to indicate that the source of these protons is at large L.

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INTRODUCTION

In the $L = 2$ to 5 range in the outer radiation belt, Davis, Hoffman, and Williamson (1964) find that the spectra of the relatively stable 0.1 to 5 Mev protons show smooth but large variations with L and equatorial pitch angle, α_0 . Protons near the earth and at α_0 near 90° are more energetic than those at larger L and at smaller α_0 . The spectra are well represented by ℓ^{-E/E_0} ; E_0 varies by about a factor of 10 with L and by a factor of 2 with α_0 . In this study a model is proposed for the explanation of these spectral variations.

Kellogg (1959) first suggested that the radiation belt might be formed through magnetic disturbances in which the third adiabatic invariant of trapped particles is violated without violating the first and second invariants. Violation of the third invariant allows motion in L -space. As particles move closer to the earth they tend to gain energy with the maintenance of the 1st invariant since, for example, E/B is a constant for 90° pitch angles. So, this process can introduce acceleration of protons. Kellogg's suggestion has been adopted for this study although the mechanism for motion in L -space is unspecified. It has further been assumed that motion in L -space is rapid compared to loss and scattering processes and that the geomagnetic field is sufficiently well represented by a dipole.

Energy and Angle Variations

If the first and second adiabatic invariants of trapped particles are maintained during motion in L -space, changes in both the energy

and equatorial pitch angle can be calculated. The first invariant is:

$$\mu = \frac{E \sin^2 \alpha_0}{B_0} = \frac{EL^3 \sin^2 \alpha_0}{.312} \quad (1)$$

where B_0 is the equatorial magnetic field. The second invariant is:

$$J = m \oint v \cos \alpha \, dS \quad (2)$$

where m is the mass, v the velocity, α the local pitch angle and S is along the guiding center. The integration is over a complete north-south oscillation. For a dipole magnetic field, equation (2) is:

$$J = v L F(\alpha_0), \quad (3)$$

$$F(\alpha_0) = 4\pi r_e \int_0^{\lambda_m} \sqrt{\left[\frac{1 - \sin^2 \alpha_0 (4 - 3 \cos^2 \lambda)^{\frac{1}{2}}}{\cos^2 \lambda} \right] (4 - 3 \cos^2 \lambda) \cos \lambda} \, d\lambda$$

where r_e is the radius of the earth, λ is the latitude, and λ_m is the mirror latitude.

Since μ and J are constants, equation (1) may be divided by the square of equation (3) to give

$$L \left[\frac{\sin \alpha_0}{F(\alpha_0)} \right]^2 = \text{constant} \quad (4)$$

From this, the changes in α_0 with L can be evaluated; some results are shown in Figure 1. Two features of these results are worthy of note:

(1) changes in α_0 with L are relatively small for $L > 2.5$ and as Davis and Chang (1962) have indicated, particles diffusing inwards assume flatter helices; (2) changes in α_0 with L are independent of energy for non-relativistic particles.

These changes in α_0 with L and equation (1) may be used to find the variation in energy with L and α_0 . Results are shown in Figure 2 for protons having α_0 values at $L = 7$ as indicated on the curves. Energies are relative to energies at $L = 7$.

Comparison of Spectra with Experimental Results

The spectra of protons that one would expect at some L and α_0 depends on the location and nature of the source and on the energy dependence of motion in L -space. The source is assumed to be at a single L and to consist of a single spectrum of protons. The result of the superposition of sources at different L and of different spectra can be obtained from the results of the above assumption. The energy dependence of motion in L -space has been examined for two processes where the third invariant only is violated. When the violation mechanism involves electric fields, the velocity of L -space motion is proportional to the vector product of the electric and the magnetic fields and does not depend on particle energy. Another process that produces violation of only the third invariant depends on asymmetric distortions of the geomagnetic field such as occur with sudden commencements and sudden impulses (Parker, 1960). Motion in L -space for this process depends on the guiding center of particles following

magnetic field lines during rapid changes in the field and is also independent of energy. Thus, motion in L-space, when only the third invariant is violated, appears to be independent of energy.

With the above results and assumptions and the results and assumption of the previous sections, changes in spectra for motion in L-space are readily obtained. If the injection spectrum is power law, the spectrum remains power law with the same exponent. If the injection spectrum has an exponential form, e^{-E/E_0} , the spectrum remains exponential after L-space motion and E_0 varies in the same way with L and α_0 as has been calculated for a single particle in the previous section.

These two predictions of the model may be compared with experiment. The first prediction, that the spectra retains its exponential form, is in agreement with experiment. To test the second prediction, measured E_0 (Davis et. al., 1964) have been plotted in Figure 3 as a function of L with appropriate changes in α_0 with L. The labels on the curves refer to α_0 values at $L = 7$. The dashed curves in Figure 3 are taken from Figure 2 for corresponding changes in E with L and α_0 . This comparison, too, shows good agreement between the model and experimental results.

If the dashed curves in Figure 3 are extended, they intersect near $L = 10$. This intersection is the L value where the spectrum is independent of α_0 and may be interpreted as the source location.

Comparison with Liouville's Theorem

Thus far, only the spectra of particles has been compared with the model. In this section, the fluxes in L-space are compared with

predictions of Liouville's Theorem to see what might be learned about mechanisms for L-space motion. With motion in L-space, the fluxes may (1) obey Liouville's Theorem; (2) obey Liouville's Theorem but be altered by loss processes; (3) not obey Liouville's Theorem. The third possibility could occur, for example, if motion in L-space is due to diffusion.

The measured directional fluxes, $j(L, E, \alpha_0)$, have been given in units of $\text{no/cm}^2 \text{ sec ster Mev}$ (Davis, et. al., 1964). If Liouville's Theorem were to hold, the conserved quantity is j/E . In the comparison between measurements and Liouville's theorem both E and α_0 are varied with L as is appropriate for following the trajectory of a particle when the first two adiabatic invariants are maintained.

Figure 4 shows results which are typical of the experimental data. Since j/E is not conserved, the first possibility above is ruled out.

An interesting feature of Figure 4 is the fact that j/E increases monotonically with L . This also indicates that the source is at large L .

Conclusions

In this study a rather simple model has been found to successfully explain spectral changes with L and α_0 of the protons in the outer radiation belt. The trends in the variations in the spectra and in comparison of fluxes with Liouville's Theorem both indicate that the

source is near the edge of the magnetosphere. According to this model, the protons are moved within the outer radiation belt and accelerated by some mechanism that violates the third adiabatic invariant of charged particle motion without violating the first two invariants. Since fluxes do not obey Liouville's Theorem, the mechanism is of the diffusion type or one where losses must be considered along with the motion where fluxes obey Liouville's Theorem.

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Figure Captions

Figure 1 - Variation in equatorial pitch angle with L when μ and J are constant.

Figure 2 - Relative variation in proton energy with L when μ and J are constant.

Figure 3 - Comparison between measured and predicted variations in E_0 with L and α_0 .

Figure 4 - Relative variation of j/E with L.

